NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

Level 3, 2004

Statistics and Modelling

Calculate confidence intervals for population parameters (90642)

Solve straightforward problems involving probability (90643)

Solve equations (90644)

Use probability distribution models to solve straightforward problems (90646)

National Statistics

Assessment Report

Assessment Schedule

Statistics and Modelling Level 3, 2004

General Comments

Candidates gaining Achievement demonstrated that they had the mathematical skills to meet requirements identified in the standards being assessed.

In many scripts candidates rounded answers prematurely, or did not round a final answer to a degree of accuracy that was appropriate to the problem. While schedules allowed for a reasonable variation in rounding, particularly at Achieved level, candidates should be encouraged to provide answers to a sensible degree of accuracy.

In many cases candidates could gain Achieved with answer only or responses written in different forms. However, there are still situations where they should be encouraged to write answers and show working in forms that have been traditionally accepted in mathematics education. The way confidence intervals are written, and the use of shaded diagrams in normal distribution calculations, would be examples of this.

Candidates who gained Achieved were expected to have an understanding of standard mathematical notations. The understanding of the use of *A* as the complement of *A* would be an example of this.

Questions that required candidates to “justify”, “give mathematical reasons” or similar provided a great variety in terms of responses. Candidates should be encouraged to identify conditions, then clearly and precisely communicate the mathematics of the response. Extended written answers were not expected for such questions.

Candidates should be encouraged to attempt all questions in a paper as the use of replacement evidence provides multiple opportunities to reach the Achieved level.

Candidates should be aware of the need to clearly cross out any work that is not to be marked.

Statistics and Modelling: Calculate confidence intervals for population parameters (90642)

National Statistics

Number of	Percentage achieved			
Results	Not Achieved	Achieved	Merit	Excellence
11683	11.9	36.1	47.4	4.6

Assessment Report

Candidates gaining Achievement were able to understand the basic concepts of confidence intervals, were able to calculate correct *z*-values and were able to substitute into appropriate formulae.

Candidates should be encouraged to write confidence answers in a suitable form; for example $1195 < \mu < 1371$.

Premature rounding had a huge effect on confidence interval answers, and should be avoided. The rounding of final answers to a sensible degree of accuracy should be encouraged.

Grades above the Achieved level required an understanding of confidence intervals. Candidates needed to demonstrate the ability to precisely communicate statistical understanding, and to have skills in algebraic manipulation in order to reach higher levels of understanding. When establishing minimum sample sizes candidates need to express their final answer as a single number, not an inequality.

Statistics and Modelling: Solve straightforward problems involving probability (90643)

National Statistics

Number of Results	Percentage achieved			
	Not Achieved	Achieved	Merit	Excellence
11738	44.8	40.2	11.4	3.7

Assessment Report

Candidates gaining Achievement demonstrated an understanding of a range of probability skills and were able to select appropriate techniques to solve probability problems. Successful candidates were able to manipulate fractions and correctly substitute into appropriate formulae.

Many candidates demonstrated a lack of understanding of basic probability theory and lacked logic in the presentation of answers.

Some candidates obtained Achieved by successfully answering Merit level questions, sometimes using methods that did not involve probability theory.

Candidates should be encouraged to understand, and use, accepted probability notation, particularly basic set theory.

Some candidates had difficulty in justifying a conclusion that events were not independent. A clear understanding of the definition was expected for this.

Candidates who successfully completed the expected value question understood that a negative expectation in dollar terms is a loss.

Statistics and Modelling: Solve equations (90644)

National Statistics

Number of Results	Percentage achieved			
	Not Achieved	Achieved	Merit	Excellence
11618	28.8	50.6	10.1	8.1

Assessment Report

Candidates gaining Achievement were able to solve linear systems of equations, solve linear programming problems and solve non-linear equations.

Successful candidates understood key words (for example “iterate”) and realised that, in some questions, clear systematic working was helpful in obtaining a correct solution.

For questions relating to systems of simultaneous equations, successful candidates either systematically set their answers out or used their graphics calculator correctly. The checking of final answers should be encouraged. Candidates who achieved at higher levels understood contextual questions and distinguished between algebraic and geometric solutions to systems of equations.

For linear programming questions, successful candidates were accurate with the drawing of lines, correctly identified a feasible region and tested all vertices of the region. A few candidates used the “parallel line” test in

obtaining an optimal solution. Candidates who did not attempt the Merit level question lost a possible replacement evidence opportunity.

For non-linear equations, successful candidates showed the results of each iteration, sometimes completing more than the required number of iterations. Most candidates used the Newton-Raphson method for solving equations. Some candidates who attempted the Merit level question failed to give their final answer to the required accuracy.

For students without a graphics calculator, this standard requires considerable effort and mental energy in solving equations. Candidates with a graphics calculator had a distinct advantage.

Statistics and Modelling: Use probability distribution models to solve straightforward problems (90646)

National Statistics

Number of Results	Percentage achieved			
	Not Achieved	Achieved	Merit	Excellence
11712	31.5	39.9	22.1	6.5

Assessment Report

Candidates gaining Achieved were able to solve problems using the binomial, Poisson and normal distributions.

Candidates were expected to recognise which distribution was required and identify parameters. Successful candidates were able to recognise the first question as a binomial one, calculate a correct λ value in the second question and convert times to a decimal form in the third question.

Questions of the form "more than 4" needed to be correctly interpreted, and a logical presentation of answers, particularly with the normal distribution, should be encouraged. When reading tables candidates should ensure that they are reading values from the correct row or column.

Candidates gaining grades above the Achieved level demonstrated the ability to find the probability of combinations of events and linear combinations of variables. While the final answer to the Excellence question could be completed using a graphics calculator, candidates still needed to be able to identify, and justify, the use of an approximating distribution.

Assessment Schedule
Statistics and Modelling: Calculate confidence intervals for population parameters (90642)

	Assessment Criteria	No.	Evidence	Code	Judgement	Sufficiency
Achievement	Calculate confidence intervals for population parameters.	1(a) 1(b) 1(c)	1283 ± 88 or $1195 < \mu < 1371$ 0.19 ± 0.07 or $0.12 < \pi < 0.26$ 15.85 ± 1.99 or 1.98 $13.86 < \mu < 17.84$ or $13.87 < \mu < 17.83$	A A A	Correct answer only. Accept any rounding more than 1 sig. fig. for all 3 intervals. Accept intervals written in equivalent forms, focus on the correct numbers.	**Achievement** Two of Code A.
Merit	Demonstrate an understanding of confidence intervals.	1(d)	$n = 62$ or $n \geq 62$	A M	Must round up to 62. $n > 62$, $n = 61$ or $n > 61.46\ldots$ gets an A. Using error = 10, so $n = 246$ gets an A.	**Merit** Achievement plus two of code M **or** three of code M.
		2(a)	$\text{SE} = \sqrt{\frac{6.53^2}{80} + \frac{7.66^2}{95}}$ $= 1.073$ Interval is: 2.25 ± 2.10 or $0.15 < \mu_1 - \mu_2 < 4.35$	A M	Accept any rounding more than 1 sig. fig. for the interval. Accept interval written in equivalent interval notation forms, but must be in the correct order (incorrect order gives A).	
		2(b)	Results suggest that there **is a difference** because **0 does not lie in the interval**.	M	Or equivalent. Must be consistent with their interval in 2(a) and not vague. If 2(a) is incorrect then allow consistent error in 2(b).	

Excellence	Analyse estimates of population parameters.	3(a)	$n = 270$ or $n = 271$	A M E	No other alternative. $n \geq 270$ or $n \geq 271 \Rightarrow$ M Students using a numerical approach must use correct z value (99% CI) and σ ($\sigma \approx 76.5$).	**Excellence:** Merit plus two of code E.
		3(b)	$n = 30k^2$	A M	Formulae only, can provide evidence for A or M.	
			eg: SE (pilot) = $\frac{\sigma}{\sqrt{30}}$ SE (final) = $\frac{\sigma}{\sqrt{n}}$ SE (final) = $\frac{1}{k}$ SE(pilot) $\frac{\sigma}{\sqrt{30}} = \frac{1}{k}\frac{\sigma}{\sqrt{n}}$ $\sqrt{n} = k\sqrt{30}$ $n = 30k^2$	E	Must have formula correct (accept different forms), and reasoning that recognises the effect of the square root in the standard error (SE) formula. $n = 29.9\ldots k^2$ is acceptable if supported by mathematical reasoning that uses numerical values for z (99% CI) and σ ($\sigma \approx 76.5$).	

Assessment Schedule
Statistics and Modelling: Solve straightforward problems involving probability (90643)

	Assessment Criteria	No.	Evidence	Code	Judgement	Sufficiency
Achievement	Solve straight-forward problems involving probability.	1	$P(W=1)=\frac{7}{10}=0.7$ $P(W=2)=\frac{2}{15}=0.1\dot{3}$	A	Both required. Accept equivalent.	**Achievement** Three of Code A.
		2	$\frac{3}{4}=0.75$	A	Accept equivalent.	
		3(a)	No, as $P(B) \times P(T) \neq P(B \cap T)$ $0.0425 \neq 0.045$ or $0.25 \times 0.17 \neq 0.045$	A	Justification needed. Need to have stated 'not independent' in some form and have correct numerical justification.	
		3(b)	0.625	A	Accept equivalent.	
Merit	Solve probability problems.	3(c)	0.265 (3 s.f.) or 0.2647, 0.26	A M	Accept equivalent.	**Achievement with Merit:** Achievement plus three of Code M **or** four of Code M.
		4	–\$2.50 or a loss of \$2.50 States $E(x)=\frac{-1}{6}$	A M A	Accept equivalent.	
		5	$\frac{45}{200}=\frac{9}{40}=0.225$	A M	Accept equivalent.	
		6	$\frac{1050}{2380}=\frac{15}{34}=0.441$ (3 s.f.)	A M	Accept equivalent.	
Excellence	Apply probability theory.	7	Girl (T1) × Girl (T2) + Boy (T1) × Girl (T2) $=\frac{y\times(w+1)}{(y+z)(w+x+1)}+\frac{z\times w}{(y+z)(w+x+1)}$ Correct tree diagram only	A M E A	Tree diagram or equivalent plus justification needed. Justification is statement as given, words to that effect or a correctly drawn tree diagram. Must show addition of probabilities.	**Achievement with Excellence:** Merit plus Code E.

Assessment Schedule
Statistics and Modelling: Solve equations (90644)

	Assessment Criteria	No.	Evidence	Code	Judgement	Sufficiency
Achievement	Solve equations.	1(a)	$x = -1$ $y = -4$ $z = 5$	A1	No alternative.	**Achievement** Three of Code A (must have at least one Code A1 and at least one Code A2).
		1(b)	$a = 7$ $b = 2.5$ $c = 3$	A1	No alternative.	
		2	Newton-Raphson method with starting value 0.5 gives iterates 0.1477 and 0.1668. Bisection method with starting values [0,1] gives: First iteration $f(0) = 1$, $f(1) = -4$, $f(0.5) = -1.9375$ giving a new interval of [0,0.5]. Second iteration $f(0.25) = -0.4961$, giving a new interval of [0, 0.25] or an approximate root of 0.125.	A2	Accept either Newton Raphson or Bisection method. Must show evidence of getting iterates – eg $f'(x)$ or calculations shown – eg signs of $f'(x)$ or mid-points of intervals shown. Accept with other starting values.	
		3(a) 3(b) (i) (ii)	Q 3(a) need not be completed to answer the question. Must calculate return at the vertices (0,320), (600,120), (720,0) or use the "parallel line test" to identify the optimal vertex. $s = 600$ and $b = 120$. Maximum return $R = \$2400$	A2	Need max. return and how many bags	

<table>
<tr><th></th><th>Assessment Criteria</th><th>No.</th><th>Evidence</th><th>Code</th><th>Judgement</th><th>Sufficiency</th></tr>
<tr><td rowspan="4">Merit</td><td rowspan="4">Solve problems involving equations.</td><td>4(a)</td><td>The constraints are:
$100a + 100b \leq 60\,000$
$50a \leq 25\,000$
$50a + 100b \leq 45\,000$
a and b non-negative.
Calculate R at the vertices (0,450), (300,300) and (500,100), or use the “parallel line test” to identify the optimal vertex.
Maximum return R = \$1420 when a = 500 and b = 100.</td><td>A2
M2</td><td>Must state the number of each type of bag that gives the maximum and the maximum value.
Must clearly state maximum R.</td><td rowspan="4">Achievement with Merit:
Achievement plus three of Code M (must have at least one Code M1 and at least one Code M2).
or
Four of Code M.</td></tr>
<tr><td>5(a)</td><td>Form the system of equations
$a = b + c + 1\,600$
$b = 2c$
$a + b + c = 100\,000$
and solve to obtain
$a = 50\,800$
$b = 32\,800$
$c = 16\,400$</td><td>A1
M1</td><td>Must form equations and solve.
Accept equivalent equations.</td></tr>
<tr><td>5(b)</td><td>Form the system of equations
$c + b + f = 740$
$2.5c + 3b + 2f = 1935$
$150c + 200b + 120f = 122\,500$
and solve to obtain
$c = 270$
$b = 320$
$f = 150$</td><td>A1
M1</td><td>Must form equations and solve.
Accept equivalent equations.</td></tr>
<tr><td>6</td><td>10.3
Newton-Raphson method, with any starting value between 10 and 11, gives a solution of 10.3 from 2 iterations.
Bisection method starting with [10,11] gives a solution of 10.3 from 4 iterations.</td><td>A2
M2</td><td>Must be rounded to 1 d.p.
Accept one minor arithmetic error.
Accept other starting values.
Must show evidence of getting iterates
– eg $f'(x)$ or calculations shown
– eg signs of $f'(x)$ or mid-points of intervals shown.</td></tr>
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	Assessment Criteria	No.	Evidence	Code	Judgement	Sufficiency
Excellence	Analyse or interpret the outcome of the process used to solve the equations or linear programming problems.	4(b)	$300 \le a \le 500$ and $a + b = 600$.	M2 E	Or equivalent in words or symbols.	**Achievement with Excellence:** Merit plus two Code E
		7	Two planes are parallel and the third plane intersects the other two planes.	E	Or equivalent in words or diagram.	
		8(a)	Tangents drawn, and x_1 and x_2 consistently indicated on the graph.	E	8(a) and 8(b) are both needed. Accept answer using $x_0 = 0.5$ or $x_0 = 1.5$.	
		8(b)	A statement that either identifies that 1.5 will produce a sequence of iterates that converges to another root, or recognises the effect of the turning point. Must support with some reasoning, eg about the gradient of the tangents.			

Assessment Schedule
Statistics and Modelling: Use probability distribution models to solve straightforward problems (90646)

	Achievement Criteria	No.	Evidence	Code	Judgement	Sufficiency
Achievement	Use probability distribution models to solve straight-forward problems.	1	Binomial distribution P ($X > 4$, $n = 6$, $\pi = 0.4$) = 0.0369 + 0.0041 = 0.041	A	Or equivalent.	**Achievement:** Two of Code A.
		2	Poisson distribution P ($X = 0$, $\lambda = 2$) = 0.135	A	Or equivalent.	
		3(a)	Normal distribution P ($-20 < X < 40$) or P ($0 < X < 60$) = P ($-1.111 < Z < 2.222$) = 0.3667 + 0.4869 = 0.8536	A	Or equivalent.	
Merit	Use probability distribution models to solve problems.	3(b)	Normal distribution $Z = 0.841$ $X = 20 + 0.841 \times 18$ $= 35.14$ or 35.15 At about 10:35 am	A M	Or equivalent.	**Merit:** Achievement **plus** two of Code M **or** three of Code M.
		4	Normal distribution P ($X < 300$) = P ($Z < -1$) = 0.5 – 0.3413 = 0.1587	A	Or equivalent.	
			P (Both underweight) $= 0.1587^2$ = 0.0252	M	Or equivalent.	
		5	$E(T) = 12 \times 64 = 768$ $\sigma(T) = \sqrt{12 \times 3^2}$ $= 10.392$ Normal distribution P ($T \geq 750$) = P ($Z \geq -1.732$) = 0.5 + 0.4584 = 0.9584	A M	Or equivalent.	

	Achievement Criteria	No.	Evidence	Code	Judgement	Sufficiency
Excellence	Use and justify probability distribution models to solve complex problems.	6(a)	Binomial because • Fixed number of trials $n = 150$ • Probability remains constant $\pi = 0.9$ or $\pi = 0.1$ • Two possible outcomes, a paling is either bent or not bent. • Independence stated.	E	Must state distribution Need 3/4 criteria listed with qualifying details from problem. Or equivalent.	**Excellence:** Merit **plus** three of Code E
		6(b)	Normal approximation to binomial appropriate because $n\pi = 135$, and $n(1 - \pi) = 15$, are both ≥ 5.	E	Need both $n\pi$ and $n(1 - \pi)$ in justification. Calculation can be implied in (a) or (c).	
		6(c)	Binomial distribution $P(X \geq 130, n = 150, \pi = 0.9)$ **Normal approximation** $= P(X \geq 129.5)$ with continuity correction (cc). $= P\left(Z \geq \dfrac{129.5 - 135}{3.674}\right)$ $= P(Z \geq -1.497)$ $= 0.5 + 0.4329$ $= 0.9329$ Accept use of Graphics calculator to find binomial probability. $P(X \geq 130, n = 150, \pi = 0.9)$ $= 1 - 0.072088$ $= 0.9279$	A M E	Use of continuity correction can be implied. Or equivalent. No use of continuity correction gives M $P(X \geq 130) = 0.913$ Shown by: $\wedge$cc M	